

N=2 $SO(4)$ 7D gauged supergravity with topological mass term from 11 dimensions

Parinya Karndumri

*String Theory and Supergravity Group, Department of Physics, Faculty of Science,
Chulalongkorn University, 254 Phayathai Road, Pathumwan, Bangkok 10330,
Thailand*

E-mail: parinya.ka@hotmail.com

ABSTRACT: We construct a consistent reduction ansatz of eleven-dimensional supergravity to $N = 2$ $SO(4)$ seven-dimensional gauged supergravity with topological mass term for the three-form field. The ansatz is obtained from a truncation of the S^4 reduction giving rise to the maximal $N = 4$ $SO(5)$ gauged supergravity. Therefore, the consistency is guaranteed by the consistency of the S^4 reduction. Unlike the gauged supergravity without topological mass having a half-supersymmetric domain wall vacuum, the resulting 7D gauged supergravity theory admits a maximally supersymmetric AdS_7 critical point. This corresponds to $N = (1, 0)$ superconformal field theory in six dimensions. We also study RG flows from this $N = (1, 0)$ SCFT to non-conformal $N = (1, 0)$ Super Yang-Mills theories in the seven-dimensional framework and use the reduction ansatz to uplift this RG flow to eleven dimensions.

KEYWORDS: AdS-CFT correspondence, Gauge/Gravity Correspondence and Supergravity Models.

1. Introduction

Gauged supergravities in various dimensions play an important role in both string compactifications and in the AdS/CFT correspondence. In some cases, a consistent truncation can be made in such a way that a lower dimensional gauged supergravity is obtained via a dimensional reduction of a (gauged) supergravity in higher dimensions on spheres [1]. Embedding lower dimensional gauged supergravities is now of considerable interest since this provides a method to uplift lower dimensional solutions to string/M theory.

It is known that sphere reductions of 10 or 11 dimensional supergravities give rise to gauged supergravity in lower dimensions. Well-known examples of these consistent sphere reductions include S^7 and S^4 reductions of eleven-dimensional supergravity and S^5 reduction of type IIB theory giving rise to $SO(8)$, $SO(5)$ and $SO(6)$ gauged supergravities in four, seven and five dimensions, respectively [2, 3, 4]. According to the AdS/CFT correspondence [5], seven-dimensional gauged supergravity is useful in the study of $N = (2, 0)$ and $N = (1, 0)$ field theories in six dimensions [6, 7, 8, 9, 10]. The latter describe the dynamics of M5-branes worldvolume in M-theory and are less-known on the field theory side. Therefore, seven-dimensional gauged supergravity is expected to give some insight to six-dimensional field theories via gauge/gravity correspondence.

In this paper, we are interested in obtaining $N = 2$ seven-dimensional gauged supergravity with $SO(4)$ gauged group and topological mass term. In seven dimensions, the theory is obtained by coupling three vector multiplets to the pure $SU(2)$ gauged supergravity constructed in [11]. This matter-coupled theory has been constructed in [12] and [13]. The $SO(4)$ gauged supergravity has also been constructed in [14] by truncating the maximal $N = 4$ $SO(5)$ gauged supergravity. All of these constructions have not included the topological mass term for the three-form field, and the resulting theory does not admit AdS_7 vacuum solutions. It has been shown in [15] that the topological mass term is possible. The massive gauged theory has been explored in [16] in which new AdS_7 vacua and the corresponding RG flow interpolating between these vacua have been given.

To give an interpretation to this solution in the string/M theory context, it is necessary to embed this solution to 10 or 11 dimensions. The reduction ansatz of eleven-dimensional supergravity giving rise to pure $SU(2)$ gauged supergravity has been given in [17]. The $SO(4)$ gauged theory without topological mass term from a dimensional reduction of eleven- and ten-dimensional supergravity has been given in [18] using the result of [19]. This result is clearly not sufficient to uplift the solution in [16]. The dimensionally reduced theory needs to include the topological mass term in order to admit AdS_7 vacua. We will give an extension to the result of [17, 18] by constructing

$SO(4)$ gauged theory including topological mass term from a truncation of S^4 reduction of eleven dimensional supergravity. This provides an ansatz to uplift the 7-dimensional solutions of massive $N = 2$ $SO(4)$ gauged supergravity to eleven dimensions.

The paper is organized as follow. In section 2, we give relevant formulae for $N = 2$ $SO(4)$ gauged supergravity in seven dimensions. The embedding of this theory in eleven dimensions is obtained via a consistent truncation of the S^4 reduction of eleven-dimensional supergravity in section 3. We then use the resulting ansatz to uplift RG flow solutions from the maximally supersymmetric AdS_7 vacuum with $SO(4)$ symmetry to non-conformal SYM in section 4. We end the paper by giving some conclusions and comments in section 5.

2. $SO(4)$ $N = 2$ gauged supergravity in seven dimensions

In this section, we give a description of $SO(4)$ $N = 2$ gauged supergravity in seven dimensions with topological mass term. All of the notations are the same as those in [15] to which the reader is referred for further details.

The $SO(4)$ gauged theory is obtained by coupling three vector multiplets to the $N = 2$ supergravity multiplet. The field contents are given respectively by

$$\begin{aligned} \text{Supergravity multiplet :} & \quad (e_\mu^a, \psi_\mu^A, A_\mu^i, \chi^A, B_{\mu\nu}, \sigma) \\ \text{Vector multiplets :} & \quad (A_\mu, \lambda^A, \phi^i)^r \end{aligned} \tag{2.1}$$

where an index $r = 1, 2, 3$ labels the three vector multiplets. Curved and flat space-time indices are denoted by μ, ν, \dots and a, b, \dots , respectively. $B_{\mu\nu}$ and σ are a two-form and the dilaton fields. The two-form field will be dualized to a three-form field $C_{\mu\nu\rho}$. Indices $i, j = 1, 2, 3$ label triplets of $SU(2)_R$. The 9 scalars ϕ^{ir} are parametrized by $SO(3, 3)/SO(3) \times SO(3) \sim SL(4, \mathbb{R})/SO(4)$ coset manifold. The corresponding coset representative of $SO(3, 3)/SO(3) \times SO(3)$ will be denoted by

$$L = (L_I^i, L_I^r), \quad I = 1, \dots, 6. \tag{2.2}$$

whose inverse is given by $L^{-1} = (L^I_i, L^I_r)$ where $L^I_i = \eta^{IJ} L_{Ji}$ and $L^I_r = \eta^{IJ} L_{Jr}$. Indices i, j and r, s are raised and lowered by δ_{ij} and δ_{rs} , respectively while the full $SO(3, 3)$ indices I, J are raised and lowered by $\eta_{IJ} = \text{diag}(- - - + + +)$.

The $SO(4) \sim SU(2) \times SU(2)$ gauging is implemented by promoting the $SU(2) \times SU(2) \sim SO(3) \times SO(3) \subset SO(3, 3)$ to a gauge symmetry. The structure constants for the $SU(2) \times SU(2)$ gauge group, which will appear in various quantities, are given by

$$f_{IJK} = (g_1 \epsilon_{ijk}, g_2 \epsilon_{rst}). \tag{2.3}$$

To obtain $SO(4)$ gauge group, we will later set $g_2 = g_1$. The bosonic Lagrangian can be written in a form language as

$$\begin{aligned}\mathcal{L} = & \frac{1}{2}R * \mathbb{I} - \frac{1}{2}e^\sigma a_{IJ} * F_{(2)}^I \wedge F_{(2)}^J - \frac{1}{2} * H_{(4)} \wedge H_{(4)} - \frac{5}{8} * d\sigma \wedge d\sigma \\ & - \frac{1}{2} * P^{ir} \wedge P_{ir} + \frac{1}{\sqrt{2}}H_{(4)} \wedge \omega_{(3)} - 4hH_{(4)} \wedge C_{(3)} - V * \mathbb{I}\end{aligned}\quad (2.4)$$

where the scalar potential is given by

$$V = \frac{1}{4}e^{-\sigma} \left(C^{ir}C_{ir} - \frac{1}{9}C^2 \right) + 16h^2e^{4\sigma} - \frac{4\sqrt{2}}{3}he^{\frac{3\sigma}{2}}C. \quad (2.5)$$

The constant h describes the topological mass term for the three-form $C_{(3)}$ with $H_{(4)} = dC_{(3)}$. The quantities appearing in the above Lagrangian are defined by

$$\begin{aligned}P_\mu^{ir} &= L^{Ir} (\delta_I^K \partial_\mu + f_{IJ}{}^K A_\mu^J) L^i{}_K, & C_{rsi} &= f_{IJ}{}^K L_r^I L_s^J L_{Ki}, \\ C_{ir} &= \frac{1}{\sqrt{2}} f_{IJ}{}^K L_J^I L_K^J L_{ir} \epsilon^{ijk}, & C &= -\frac{1}{\sqrt{2}} f_{IJ}{}^K L_i^I L_j^J L_{Kk} \epsilon^{ijk}, \\ a_{IJ} &= L^i{}_I L_{iJ} + L^r{}_I L_{rJ}.\end{aligned}\quad (2.6)$$

The Chern-Simons three-form satisfying $d\omega_{(3)} = F_{(2)}^I \wedge F_{(2)}^I$ is given by

$$\omega_{(3)} = F_{(2)}^I \wedge A_{(1)}^I - \frac{1}{6} f_{IJ}{}^K A_{(1)}^I \wedge A_{(1)}^J \wedge A_{(1)K} \quad (2.7)$$

with $F_{(2)}^I = dA_{(1)}^I + \frac{1}{2} f_{JK}{}^I A_{(1)}^J \wedge A_{(1)}^K$

It is also useful to give the corresponding field equations

$$d(e^{-2\sigma} * H_{(4)}) + 16hH_{(4)} - \frac{1}{\sqrt{2}}F_{(2)}^I \wedge F_{(2)}^I = 0, \quad (2.8)$$

$$\begin{aligned}& \frac{5}{4}d * d\sigma - \frac{1}{2}e^\sigma a_{IJ} * F_{(2)}^I \wedge F_{(2)}^J + e^{-2\sigma} * H_{(4)} \wedge H_{(4)} \\ & + \left[\frac{1}{4} \left(C^{ir}C_{ir} - \frac{1}{2}C^2 \right) + 2\sqrt{2}he^{\frac{3}{2}\sigma}C - 64h^2e^{4\sigma} \right] \epsilon_{(7)} = 0\end{aligned}\quad (2.9)$$

$$D(e^\sigma a_{IJ} * F_{(2)}^I) + \frac{1}{\sqrt{2}}H_{(4)} \wedge F_{(2)}^J + *P^{ir} f_{IJ}{}^K L_r^I L_{iK} = 0 \quad (2.10)$$

$$\begin{aligned}& D * P^{ir} - e^\sigma L^i{}_I L^r{}_J * F_{(2)}^I \wedge F_{(2)}^J \\ & - * \mathbb{I} \left[\sqrt{2}e^{-\sigma} C_{jr} C^{rsk} \epsilon^{ijk} + 4\sqrt{2}he^{\frac{3\sigma}{2}} C_{ir} \right] = 0.\end{aligned}\quad (2.11)$$

The Yang-Mills equation (2.10) can be written in terms of C^{ir} and C^{irs} by using the relation

$$f_{IJ}{}^K L_r^I L_{iK} = -\frac{1}{2\sqrt{2}} \epsilon^{ijk} C^{jr} L^k{}_J - C^{irs} L_{sJ}. \quad (2.12)$$

In obtaining the scalar equation (2.11), we have used the projections in the variations of scalars as in [12]

$$\begin{aligned}\delta L^i{}_I &= X^i{}_r L^r{}_I + X^i{}_j L^j{}_I, \\ \delta L^r{}_I &= X^{rs} L_{sI} + X^{ri} L_{iI}\end{aligned}\tag{2.13}$$

which lead to

$$\begin{aligned}\delta C^2 &= 6\sqrt{2} C C^{ir} X_{ir}, \\ \delta(C^{ir} C_{ir}) &= 4\sqrt{2} C_{ir} C^{rsj} X^k{}_s \epsilon^{ijk} - \frac{2\sqrt{2}}{3} C_{ir} C X^i{}_r.\end{aligned}\tag{2.14}$$

We finally give supersymmetry transformations for fermions with all fermionic fields vanishing. These are given by

$$\begin{aligned}\delta\psi_\mu &= 2D_\mu\epsilon - \frac{\sqrt{2}}{30} e^{-\frac{\sigma}{2}} C \gamma_\mu \epsilon - \frac{1}{240\sqrt{2}} e^{-\sigma} H_{\rho\sigma\lambda\tau} (\gamma_\mu \gamma^{\rho\sigma\lambda\tau} + 5\gamma^{\rho\sigma\lambda\tau} \gamma_\mu) \epsilon \\ &\quad - \frac{i}{20} e^{\frac{\sigma}{2}} F^i_{\rho\sigma} \sigma^i (3\gamma_\mu \gamma^{\rho\sigma} - 5\gamma^{\rho\sigma} \gamma_\mu) \epsilon - \frac{4}{5} h e^{2\sigma} \gamma_\mu \epsilon,\end{aligned}\tag{2.15}$$

$$\begin{aligned}\delta\chi &= -\frac{1}{2} \gamma^\mu \partial_\mu \sigma \epsilon - \frac{i}{10} e^{\frac{\sigma}{2}} F^i_{\mu\nu} \sigma^i \gamma^{\mu\nu} \epsilon - \frac{1}{60\sqrt{2}} e^{-\sigma} H_{\mu\nu\rho\sigma} \gamma^{\mu\nu\rho\sigma} \epsilon \\ &\quad + \frac{\sqrt{2}}{30} e^{-\frac{\sigma}{2}} C \epsilon - \frac{16}{5} e^{2\sigma} h \epsilon,\end{aligned}\tag{2.16}$$

$$\delta\lambda^r = -i\gamma^\mu P^ir_\mu \sigma^i \epsilon - \frac{1}{2} e^{\frac{\sigma}{2}} F^r_{\mu\nu} \gamma^{\mu\nu} \epsilon - \frac{i}{\sqrt{2}} e^{-\frac{\sigma}{2}} C^{ir} \sigma^i \epsilon\tag{2.17}$$

where $SU(2)_R$ doublet indices A, B, \dots on spinors are suppressed. σ^i are the usual Pauli matrices.

3. Seven dimensional $N = 2$ gauged supergravity from eleven dimensions

We now construct a reduction ansatz for embedding $SO(4)$ $N = 2$ gauged supergravity mentioned in the previous section in eleven dimensions. The ansatz will be obtained from a consistent truncation of the S^4 reduction of eleven-dimensional supergravity giving rise to the maximal $N = 4$ $SO(5)$ gauged supergravity in seven dimensions. To obtain the topological mass term, we will impose the so-called odd-dimensional self-duality as in [17].

3.1 $N = 4$ $SO(5)$ gauged supergravity from seven dimensions

To set up the notations and make the paper self-contained, we briefly repeat the S^4 reduction of eleven-dimensional supergravity [3, 20]. We will work in the notations of [19] and deal mainly with bosonic fields. The field content of eleven-dimensional supergravity consists of the graviton \hat{g}_{MN} , gravitino $\hat{\psi}_M$ and a four-form field $\hat{F}_{(4)}$. Eleven-dimensional space-time indices are denoted by $M, N = 0, 1, \dots, 10$.

The S^4 reduction is characterized by the following ansatz

$$d\hat{s}_{11}^2 = \Delta^{\frac{1}{3}} ds_7^2 + \frac{1}{g^2} \Delta^{-\frac{2}{3}} T_{ij}^{-1} D\mu^i D\mu^j, \quad (3.1)$$

$$\begin{aligned} \hat{F}_{(4)} = & \frac{1}{4!} \epsilon_{i_1 \dots i_5} \left[\frac{4}{g^3} \Delta^{-2} \mu^m \mu^n T^{i_1 m} D T^{i_2 n} \wedge D\mu^{i_3} \wedge D\mu^{i_4} \wedge D\mu^{i_5} \right. \\ & + \frac{6}{g^2} \Delta^{-1} T^{i_5 j} \mu^j F_{(2)}^{i_1 i_2} \wedge D\mu^{i_3} \wedge D\mu^{i_4} - \frac{1}{g^3} \Delta^{-2} U \mu^{i_1} D\mu^{i_2} \wedge \dots \wedge D\mu^{i_5} \Big] \\ & - T_{ij} * S_{(3)}^i \mu^j + \frac{1}{g} S_{(3)}^i \wedge D\mu^i \end{aligned} \quad (3.2)$$

where the quantities appearing in the above equations are defined by

$$\begin{aligned} U &= 2T_{ij} T_{jk} \mu^i \mu^k - \Delta T_{ii}, & \Delta &= T_{ij} \mu^i \mu^j, & \mu^i \mu^i &= 1, \\ F_{(2)}^{ij} &= dA_{(1)}^{ij} + g A_{(1)}^{ik} \wedge A_{(1)}^{kj}, & D\mu^i &= d\mu^i + g A_{(1)}^{ij} \mu^j, \\ DT_{ij} &= dT_{ij} + g A_{(1)}^{ik} T_{kj} + g A_{(1)}^{jk} T_{ik}. \end{aligned} \quad (3.3)$$

The symmetric matrix T_{ij} , $i, j = 1, \dots, 5$ with unit determinant parametrize the $SL(5, \mathbb{R})/SO(5)$ coset manifold.

The bosonic field content of $N = 4$ gauged supergravity is given by the metric $g_{\mu\nu}$, ten vectors $A_{(1)}^{ij} = A_{(1)}^{[ij]}$ gauging the $SO(5)$ gauge group, five three-form fields $S_{(3)}^i$ and four-teen scalars T_{ij} . The corresponding field equations are given by

$$D(T_{ij} * S_{(3)}^j) = F_{(2)}^{ij} \wedge S_{(3)}^j, \quad (3.4)$$

$$H_{(4)}^i = g T_{ij} * S_{(3)}^j + \frac{1}{8} \epsilon_{ij_1 \dots j_4} F_{(2)}^{j_1 j_2} \wedge F_{(2)}^{j_3 j_4}, \quad (3.5)$$

$$\begin{aligned} D(T_{ik}^{-1} T_{jl}^{-1} * F_{(2)}^{ij}) &= -2g T_{i[k}^{-1} * D T_{l]i} - \frac{1}{2g} \epsilon_{i_1 i_2 i_3 k l} F_{(2)}^{i_1 i_2} \wedge H_{(4)}^{i_3} \\ &+ \frac{3}{2g} \delta_{i_1 i_2 k l}^{j_1 j_2 j_3 j_4} F_{(2)}^{i_1 i_2} \wedge F_{(2)}^{j_1 j_2} \wedge F_{(2)}^{j_3 j_4} - S_{(3)}^k \wedge S_{(3)}^l, \end{aligned} \quad (3.6)$$

$$\begin{aligned} D(T_{ik}^{-1} * D T_{kj}) &= 2g^2 (2T_{ik} T_{kj} - T_{kk} T_{ij}) \epsilon_{(7)} + T_{im}^{-1} T_{kl}^{-1} * F_{(2)}^{ml} \wedge F_{(2)}^{kj} \\ &+ T_{jk} * S_{(3)}^k \wedge S_{(3)}^i - \frac{1}{5} \delta_{ij} [2g^2 (2T_{kl} T_{kl} - (T_{kk})^2) \epsilon_{(7)} \\ &+ T_{nm}^{-1} T_{kl}^{-1} * F_{(2)}^{ml} \wedge F_{(2)}^{kn} + T_{kl} * S_{(3)}^k \wedge S_{(3)}^l] \end{aligned} \quad (3.7)$$

where

$$H_{(4)}^i = DS_{(3)}^i = dS_{(3)}^i + gA_{(1)}^{ij} \wedge S_{(3)}^j. \quad (3.8)$$

All of these equation can be obtained from the Lagrangian

$$\begin{aligned} \mathcal{L}_7 = & R * \mathbb{I} - \frac{1}{4} T_{ij}^{-1} * DT_{jk} \wedge T_{kl}^{-1} DT_{li} - \frac{1}{4} T_{ik}^{-1} T_{jl}^{-1} * F_{(2)}^{ij} \wedge F_{(2)}^{kl} - \frac{1}{4} T_{ij} * S_{(3)}^i \wedge S_{(3)}^j \\ & + \frac{1}{2g} S_{(3)}^i \wedge H_{(4)}^i - \frac{1}{8g} \epsilon_{ij_1 \dots j_4} S_{(3)}^i \wedge F_{(2)}^{j_1 j_2} \wedge F_{(2)}^{j_3 j_4} + \frac{1}{g} \Omega_{(7)} - V * \mathbb{I} \end{aligned} \quad (3.9)$$

where $\Omega_{(7)}$ is the Chern-Simons three-form whose explicit form can be found in [22]. The scalar potential for T_{ij} is given by

$$V = g^2 \left(T_{ij} T_{ij} - \frac{1}{2} (T_{ii})^2 \right). \quad (3.10)$$

We have not given Einstein equation since we will not consider Einstein equation in this paper. The consistency of the full truncation, including the Einstein equation, to $N = 2$ $SO(4)$ gauged supergravity is guaranteed from the consistency of the S^4 reduction.

For completeness, we also repeat supersymmetry transformations of fermionic fields ψ_μ and $\lambda_{\hat{i}}$. Indices $\hat{i}, \hat{j} = 1, \dots, 5$ are vector indices of the composite $SO(5)_c$ symmetry. Additionally, both ψ_μ and $\lambda_{\hat{i}}$ transform as a spinor under $SO(5)_c$ with the condition $\Gamma^{\hat{i}} \lambda_{\hat{i}} = 0$, but we have omitted the $SO(5)_c$ spinor indices to make the following expressions more compact. The $SO(5)_c$ gamma matrices will be denoted by $\Gamma^{\hat{i}}$. The associated supersymmetry transformations are given by [22]

$$\begin{aligned} \delta \psi_\mu = & D_\mu \epsilon - \frac{1}{20} g T_{\hat{i}\hat{j}} \gamma_\mu \epsilon - \frac{1}{40\sqrt{2}} (\gamma_\mu^{\nu\rho} - 8\delta_\mu^\nu \gamma^\rho) F_{\nu\rho}^{\hat{i}\hat{j}} \Gamma_{\hat{i}\hat{j}} \epsilon \\ & - \frac{1}{60} \left(\gamma_\mu^{\nu\rho\sigma} - \frac{9}{2} \delta_\mu^\nu \gamma^{\rho\sigma} \right) S_{\hat{i}\nu\rho\sigma} \Gamma^{\hat{i}} \epsilon, \end{aligned} \quad (3.11)$$

$$\begin{aligned} \delta \lambda_{\hat{i}} = & \frac{1}{16\sqrt{2}} \gamma^{\mu\nu} \left(\Gamma_{\hat{k}\hat{l}} \Gamma_{\hat{i}} - \frac{1}{5} \Gamma_{\hat{i}} \Gamma_{\hat{k}\hat{l}} \right) F_{\mu\nu}^{\hat{k}\hat{l}} \epsilon + \frac{1}{2} \gamma^\mu \Gamma^{\hat{j}} P_{\mu\hat{i}\hat{j}} \epsilon \\ & - \frac{1}{120} \gamma^{\mu\nu\rho} \left(\Gamma_{\hat{i}}^{\hat{j}} - 4\delta_{\hat{i}}^{\hat{j}} \right) S_{\hat{j}\mu\nu\rho} \epsilon + \frac{1}{2} g \left(T_{\hat{i}\hat{j}} - \frac{1}{5} T_{\hat{k}\hat{k}} \delta_{\hat{i}\hat{j}} \right) \Gamma^{\hat{j}} \epsilon \end{aligned} \quad (3.12)$$

where

$$\begin{aligned} F_{(2)}^{\hat{i}\hat{j}} = & \Pi_{\hat{i}}^i \Pi_{\hat{j}}^j F_{(2)}^{ij}, \quad T_{\hat{i}\hat{j}} = (\Pi^{-1})_{\hat{i}}^i (\Pi^{-1})_{\hat{j}}^j \delta^{ij}, \\ D\epsilon = & d\epsilon + \frac{1}{4} \omega_{ab} \gamma^{ab} \epsilon + \frac{1}{4} Q_{\hat{i}\hat{j}} \Gamma^{\hat{i}\hat{j}} \epsilon, \quad T^{ij} = (\Pi^{-1})_{\hat{i}}^i (\Pi^{-1})_{\hat{j}}^j \delta^{\hat{i}\hat{j}}, \\ P_{(\hat{i}\hat{j})} + Q_{[\hat{i}\hat{j}]} = & (\Pi^{-1})_{\hat{i}}^i \left(\delta_{\hat{i}}^j d + g A_{(1)i}^j \right) \Pi_{\hat{j}}^{\hat{k}} \delta_{\hat{j}\hat{k}}, \quad S_{(3)\hat{i}} = (\Pi^{-1})_{\hat{i}}^i S_{(3)i} \end{aligned} \quad (3.13)$$

with $\Pi_{\hat{i}}^{\hat{i}}$ being the $SL(5, \mathbb{R})/SO(5)$ coset representative.

3.2 $SO(4)$ $N = 2$ gauged supergravity from S^4 reduction

We now truncate the $N = 4$ gauged supergravity to $N = 2$ theory with topological mass term for the three-form field and $SO(4)$ gauge group. In this process, the gauge group $SO(5)$ is broken to $SO(4)$. We will split the index i as $(\alpha, 5)$ with $\alpha = 1, \dots, 4$. Furthermore, we will set $T_{5\alpha}$, S^α and $F^{5\alpha}$ to zero. The S^4 coordinates μ^i will be chosen to be $\mu^i = (\cos \xi \mu^\alpha, \sin \xi)$ in which μ^α satisfy $\mu^\alpha \mu^\alpha = 1$. Similar to μ^i , μ^α are coordinates on S^3 . The scalar truncation is given by $T_{ij} = (T_{\alpha\beta}, T_{55}) = (X\tilde{T}_{\alpha\beta}, X^{-4})$ with $\tilde{T}_{\alpha\beta}$ being unimodular. The scalar field X will be related to the $N = 2$ dilaton.

With these truncations, the three-form field equations (3.4) and (3.5) become

$$D(X^{-4} * S_{(3)}^5) = 0 \quad (3.14)$$

$$dS_{(3)}^5 = gX^{-4} * S_{(3)}^5 + \frac{1}{8}\epsilon_{\alpha\beta\gamma\delta}F_{(2)}^{\alpha\beta} \wedge F_{(2)}^{\gamma\delta}. \quad (3.15)$$

We have used $\epsilon_{5\alpha\beta\gamma\delta} = \epsilon_{\alpha\beta\gamma\delta}$. From (3.14), we see that the four-form $X^{-4} * S_{(3)}^5$ is closed. We will denote it by

$$X^{-4} * S_{(3)}^5 = -F_{(4)} = -dC_{(3)} \quad (3.16)$$

or

$$S_{(3)}^5 = X^4 * F_{(4)}. \quad (3.17)$$

To satisfy equation (3.15), we impose the odd-dimensional self-duality condition

$$S_{(3)}^5 = -gC_{(3)} + \omega_{(3)} \quad (3.18)$$

or

$$X^4 * F_{(4)} = -gC_{(3)} + \omega_{(3)} \quad (3.19)$$

where $\omega_{(3)}$, satisfying $d\omega_{(3)} = \frac{1}{8}\epsilon_{\alpha\beta\gamma\delta}F_{(2)}^{\alpha\beta} \wedge F_{(2)}^{\gamma\delta}$, is the Chern-Simons term given by

$$\omega_{(3)} = \frac{1}{8}\epsilon_{\alpha\beta\gamma\delta} \left(F_{(2)}^{\alpha\beta} \wedge A_{(1)}^{\gamma\delta} - \frac{1}{3}gA_{(1)}^{\alpha\beta} \wedge A_{(1)}^{\gamma\kappa} \wedge A_{(1)}^{\kappa\delta} \right). \quad (3.20)$$

Equations for $S_{(3)}^\alpha$ are trivially satisfied.

For the Yang-Mills equations, it can be verified that setting $F_{(2)}^{5\alpha} = 0$ satisfies their field equations. For $F_{(2)}^{\alpha\beta}$, we find

$$D \left(X^{-2} \tilde{T}_{\alpha\gamma}^{-1} \tilde{T}_{\beta\delta}^{-1} * F_{(2)}^{\gamma\delta} \right) = -2g\tilde{T}_{\gamma[\alpha}^{-1} * D\tilde{T}_{\beta]\gamma} + \frac{1}{2}\epsilon_{\alpha\beta\gamma\delta}F_{(2)}^{\gamma\delta} \wedge F_{(4)} \quad (3.21)$$

where we have used the odd-dimensional self-duality condition.

We then consider scalar equations. Equations for $T_{5\alpha}$ are trivially satisfied while the T_{55} equation gives rise to the dilaton equation

$$d(X^{-1} * dX) = \frac{1}{5}X^4 * F_{(4)} \wedge F_{(4)} - \frac{1}{20}X^{-2}\tilde{T}_{\alpha\beta}^{-1}\tilde{T}_{\gamma\delta}^{-1} * F_{(2)}^{\beta\delta} \wedge F_{(2)}^{\alpha\gamma} - \frac{1}{10}g^2 \left[4X^{-8} - 3X^{-3}\tilde{T}_{\alpha\alpha} - 2X^2 \left(\tilde{T}_{\alpha\beta}\tilde{T}_{\alpha\beta} - \frac{1}{2}(\tilde{T}_{\alpha\alpha})^2 \right) \right] \epsilon_{(7)}. \quad (3.22)$$

For $T_{ij} = T_{\alpha\beta}$, we find

$$\begin{aligned} D(\tilde{T}_{\alpha\gamma}^{-1} * D\tilde{T}_{\gamma\beta}) + \delta_{\alpha\beta}d(X^{-1} * dX) &= X^{-2}\tilde{T}_{\alpha\gamma}^{-1}\tilde{T}_{\delta\kappa}^{-1} * F_{(2)}^{\gamma\kappa} \wedge F_{(2)}^{\delta\beta} \\ &+ 2g^2 \left[X^2 \left(2\tilde{T}_{\alpha\gamma}\tilde{T}_{\gamma\beta} - \tilde{T}_{\gamma\gamma}\tilde{T}_{\alpha\beta} \right) - X^{-3}\tilde{T}_{\alpha\beta} \right] \epsilon_{(7)} \\ &+ \delta_{\alpha\beta} \left[\frac{1}{5}X^4 * F_{(4)} \wedge F_{(4)} - \frac{1}{5}X^{-2}\tilde{T}_{\gamma\delta}^{-1}\tilde{T}_{\kappa\lambda}^{-1} * F_{(2)}^{\delta\lambda} \wedge F_{(2)}^{\kappa\gamma} \right. \\ &\left. - \frac{2}{5}g^2 \left[2X^2 \left(\tilde{T}_{\gamma\delta}\tilde{T}_{\gamma\delta} - \frac{1}{2}(\tilde{T}_{\gamma\gamma})^2 \right) + X^{-8} - 2X^{-3}\tilde{T}_{\gamma\gamma} \right] \epsilon_{(7)} \right]. \quad (3.23) \end{aligned}$$

We can now use the X equation (3.22) and end up with

$$\begin{aligned} D(\tilde{T}_{\alpha\gamma}^{-1} * D\tilde{T}_{\gamma\beta}) &= 2g^2 \left[2X^2 \left(\tilde{T}_{\alpha\gamma}\tilde{T}_{\gamma\beta} - \frac{1}{2}\tilde{T}_{\gamma\gamma}\tilde{T}_{\alpha\beta} \right) - X^{-3}\tilde{T}_{\alpha\beta} \right] \epsilon_{(7)} \\ &+ X^{-2}\tilde{T}_{\alpha\gamma}^{-1}\tilde{T}_{\delta\kappa}^{-1} * F_{(2)}^{\gamma\kappa} \wedge F_{(2)}^{\delta\beta} + \delta_{\alpha\beta} \left[\left\{ \frac{5}{2}g^2X^2 \left(\tilde{T}_{\gamma\delta}\tilde{T}_{\gamma\delta} - \frac{1}{2}(\tilde{T}_{\gamma\gamma})^2 \right) \right. \right. \\ &\left. \left. + \frac{1}{2}g^2X^{-3}\tilde{T}_{\gamma\gamma} \right\} \epsilon_{(7)} - \frac{1}{4}X^{-2}\tilde{T}_{\gamma\delta}^{-1}\tilde{T}_{\kappa\lambda}^{-1} * F_{(2)}^{\delta\lambda} \wedge F_{(2)}^{\kappa\gamma} \right] \quad (3.24) \end{aligned}$$

With all of the above truncations, we find the following ansatz for the metric and

the four-form field

$$\begin{aligned}
d\hat{s}_{11}^2 &= \Delta^{\frac{1}{3}} ds_7^2 + \frac{2}{g^2} \Delta^{-\frac{2}{3}} X^3 \left[X \cos^2 \xi + X^{-4} \sin^2 \xi \tilde{T}_{\alpha\beta}^{-1} \mu^\alpha \mu^\beta \right] d\xi^2 \\
&\quad - \frac{1}{g^2} \Delta^{-\frac{2}{3}} X^{-1} \tilde{T}_{\alpha\beta}^{-1} \sin \xi \mu^\alpha d\xi D\mu^\beta + \frac{1}{2g^2} \Delta^{-\frac{2}{3}} X^{-1} \tilde{T}_{\alpha\beta}^{-1} \cos^2 \xi D\mu^\alpha D\mu^\beta, \quad (3.25) \\
\hat{F}_{(4)} &= F_{(4)} \sin \xi + \frac{1}{g} X^4 \cos \xi * F_{(4)} \wedge d\xi + \frac{1}{g^3} \Delta^{-2} U \cos^3 \xi d\xi \wedge \epsilon_{(3)} \\
&\quad + \frac{1}{3!g^3} \epsilon_{\alpha\beta\gamma\delta} \Delta^{-2} X^{-3} \sin \xi \cos^4 \xi \mu^\kappa \left[5 \tilde{T}^{\alpha\kappa} X^{-1} dX + D\tilde{T}^{\alpha\kappa} \right] \wedge D\mu^\beta \wedge D\mu^\gamma \wedge D\mu^\delta \\
&\quad + \frac{1}{2g^3} \epsilon_{\alpha\beta\gamma\delta} \Delta^{-2} \cos^3 \xi \mu^\kappa \mu^\lambda \left[\cos^2 \xi X^2 \tilde{T}^{\alpha\kappa} D\tilde{T}^{\beta\lambda} - \sin^2 \xi X^{-3} \delta^{\beta\lambda} D\tilde{T}^{\alpha\kappa} \right. \\
&\quad \left. - 5 \sin^2 \xi \tilde{T}^{\alpha\kappa} X^{-4} \delta^{\beta\lambda} dX \right] \wedge D\mu^\gamma \wedge D\mu^\delta \wedge d\xi + \frac{1}{2g^2} \cos \xi \epsilon_{\alpha\beta\gamma\delta} \times \\
&\quad \left[\frac{1}{2} \cos \xi \sin \xi X^{-4} D\mu^\gamma - \left(X^{-4} \sin^2 \xi \mu^\gamma + X^2 \cos^2 \xi \tilde{T}^{\gamma\kappa} \mu^\kappa \right) d\xi \right] \wedge F_{(2)}^{\alpha\beta} \wedge D\mu^\delta \quad (3.26)
\end{aligned}$$

where

$$\begin{aligned}
U &= \sin^2 \xi \left(X^{-8} - X^{-3} \tilde{T}_{\alpha\alpha} \right) + \cos^2 \xi \mu^\alpha \mu^\beta \left(2X^2 \tilde{T}_{\alpha\gamma} \tilde{T}_{\gamma\beta} - X^2 \tilde{T}_{\alpha\beta} \tilde{T}_{\gamma\gamma} - X^{-3} \tilde{T}_{\alpha\beta} \right) \\
\epsilon_{(3)} &= \frac{1}{3!} \epsilon_{\alpha\beta\gamma\delta} \mu^\alpha D\mu^\beta \wedge D\mu^\gamma \wedge D\mu^\delta. \quad (3.27)
\end{aligned}$$

All of the above equations reduce to the pure $N = 2$ gauged supergravity with $SU(2)$ gauge group for $\tilde{T}_{\alpha\beta} = \delta_{\alpha\beta}$ after using various relations given in [21]. Note that for $\tilde{T}_{\alpha\beta} = \delta_{\alpha\beta}$, equation (3.24) gives

$$*F_{(2)}^{\alpha\gamma} \wedge F_{(2)}^{\gamma\beta} = \frac{1}{4} \delta_{\alpha\beta} *F_{(2)}^{\gamma\delta} \wedge F_{(2)}^{\delta\gamma} \quad (3.28)$$

which means that the $SO(4)$ gauge fields $A_{(1)}^{\alpha\beta}$ must be truncated to those of $SU(2)$ satisfying $F_{(2)}^{\alpha\beta} = \pm \frac{1}{2} \epsilon_{\alpha\beta\gamma\delta} F_{(2)}^{\gamma\delta}$. This is expected since there are only three vector fields in the pure gauged supergravity which only admit $SU(2)$ gauging.

The above equations can be obtained from the Lagrangian

$$\begin{aligned}
\mathcal{L}_7 &= R * \mathbb{I} - \frac{1}{4} X^{-2} \tilde{T}_{\alpha\gamma}^{-1} \tilde{T}_{\beta\delta}^{-1} * F_{(2)}^{\alpha\beta} \wedge F_{(2)}^{\gamma\delta} - \frac{1}{4} \tilde{T}_{\alpha\beta}^{-1} * D\tilde{T}_{\beta\gamma} \wedge \tilde{T}_{\gamma\delta}^{-1} D\tilde{T}_{\delta\alpha} \\
&\quad - \frac{1}{2} X^4 * F_{(4)} \wedge F_{(4)} + \frac{1}{8} \epsilon_{\alpha\beta\gamma\delta} C_{(3)} \wedge F_{(2)}^{\alpha\beta} \wedge F_{(2)}^{\gamma\delta} - 5X^{-2} * dX \wedge dX \\
&\quad - \frac{1}{2} g F_{(4)} \wedge C_{(3)} - V * \mathbb{I} \quad (3.29)
\end{aligned}$$

where the scalar potential is given by

$$V = \frac{1}{2}g^2 \left[X^{-8} - 2X^{-3}\tilde{T}_{\alpha\alpha} + 2X^2 \left(\tilde{T}_{\alpha\beta}\tilde{T}_{\alpha\beta} - \frac{1}{2}\tilde{T}_{\alpha\alpha}^2 \right) \right]. \quad (3.30)$$

For $\tilde{T}_{\alpha\beta} = \delta_{\alpha\beta}$, we find $\tilde{T}_{\alpha\alpha} = \tilde{T}_{\alpha\beta}\tilde{T}_{\alpha\beta} = 4$. The above potential becomes

$$V = \frac{1}{2}g^2 (X^{-8} - 8X^{-3} - 8X^2) \quad (3.31)$$

which is exactly the same as that given in [17] up to a redefinition of the coupling constant g .

We can also check another truncation namely to $U(1) \times U(1)$ gauged supergravity. To preserve $SO(2) \times SO(2)$ symmetry, we take the scalar matrix to be

$$\tilde{T}_{\alpha\beta} = \begin{pmatrix} e^{\frac{\phi_1}{\sqrt{2}}} & & & \\ & e^{\frac{\phi_1}{\sqrt{2}}} & & \\ & & e^{-\frac{\phi_1}{\sqrt{2}}} & \\ & & & e^{-\frac{\phi_1}{\sqrt{2}}} \end{pmatrix} \quad (3.32)$$

and define $X = e^{-\frac{\phi_2}{\sqrt{10}}}$. The potential (3.30) becomes

$$V = \frac{1}{2}g^2 \left[e^{\frac{8\phi_2}{\sqrt{10}}} - 8e^{-\frac{2\phi_2}{\sqrt{10}}} - 4e^{\frac{3\phi_2}{\sqrt{10}}} \left(e^{\frac{\phi_1}{\sqrt{2}}} + e^{-\frac{\phi_1}{\sqrt{2}}} \right) \right] \quad (3.33)$$

which takes the same form as that given in [23]. Finally, it should be remarked that the three-form field equation coming from the Lagrangian (3.29) needs to be supplemented with the odd-dimensional self-duality condition as in the pure $SU(2)$ gauged supergravity discussed in [17].

The nine scalars, parametrized by $\tilde{T}_{\alpha\beta}$, in the dimensionally reduced theory are encoded in the $SL(4, \mathbb{R})/SO(4)$ coset manifold. Therefore, in order to compare the result with gauged $N = 2$ $SO(4)$ supergravity given in the previous section, we need to use the relation between $SL(4, \mathbb{R})/SO(4)$ and $SO(3, 3)/SO(3) \times SO(3)$ coset manifolds. This is given in [15]. For the details of this mapping, the reader is referred to [15]. We will only give the $SO(3, 3)/SO(3) \times SO(3)$ coset representative $L_I^A = (L_I^i, L_I^r)$ and that of $SL(4, \mathbb{R})/SO(4)$, \mathcal{V}_R^α with $R = 1, \dots, 4$,

$$L_I^A = \frac{1}{4}\Gamma_I^{\alpha\beta}\eta_{RS}^A\mathcal{V}_\alpha^R\mathcal{V}_\beta^S \quad (3.34)$$

where Γ^I and η^A are chirally projected $SO(3, 3)$ gamma matrices.

It can be shown that the scalar potential can be written as

$$\begin{aligned} V &= \frac{1}{4}e^{-\sigma} \left(C^{ir}C_{ir} - \frac{1}{9}C^2 \right) + 16h^2e^{4\sigma} - \frac{4\sqrt{2}}{3}he^{\frac{3\sigma}{2}}C \\ &= \frac{1}{8}e^{-\sigma} \left(T_{\alpha\beta}T_{\alpha\beta} - \frac{1}{2}T_{\alpha\alpha}^2 \right) + 2T_{\alpha\alpha}he^{\frac{3\sigma}{2}} + 16h^2e^{4\sigma} \end{aligned} \quad (3.35)$$

This form is similar to the potential (3.30) if $\tilde{T}_{\alpha\beta}$ is identified with $T_{\alpha\beta}$. Note that $T_{\alpha\beta}$ and C , C^{ir} contain the gauge coupling g_1 and g_2 . In order to compare the Lagrangian of the two theories, we need to multiply the Lagrangian (2.4) by two and separate the coupling constants g_1 and g_2 from the structure constants $f_{IJK} = (g_1\epsilon_{ijk}, g_2\epsilon_{rst})$. With these, the two scalar potentials are exactly the same if we identify

$$g_2 = g_1 = -16h = -2g. \quad (3.36)$$

We also need to redefine the following fields in the Lagrangian (2.4):

$$\begin{aligned} H_{(4)} &\rightarrow \frac{F_{(4)}}{\sqrt{2}}, & C_{(3)} &\rightarrow \frac{C_{(3)}}{\sqrt{2}}, \\ F^I &= \frac{1}{4}\Gamma_{\alpha\beta}^I F_{(2)}^{\alpha\beta} & \text{or} & & F_{(2)}^{\alpha\beta} &= -\frac{1}{2}\epsilon^{\alpha\beta\gamma\delta}\Gamma_{\gamma\delta}^I F^I \\ X &= e^{-\frac{\sigma}{2}}. \end{aligned} \quad (3.37)$$

By using (3.34), it can also be checked that

$$\tilde{T}_{\alpha\gamma}^{-1}\tilde{T}_{\beta\delta}^{-1} = \frac{1}{4}\Gamma_{\alpha\beta}^I\Gamma_{\gamma\delta}^J (L^i{}_I L_{iJ} + L^r{}_I L_{rJ}). \quad (3.38)$$

The field equations from the two theories also match.

We now move to supersymmetry transformations of fermions. The maximal $N = 4$ theory contains the gravitini ψ_μ and the spin- $\frac{1}{2}$ fields λ_i . The latter is decomposed into (λ_R, λ_5) . The $SO(5)_c$ $\Gamma^{\hat{i}}$ gamma matrices are accordingly decomposed as $\Gamma^{\hat{i}} = (\Gamma^R, \Gamma^5)$. $\Gamma^5 = \Gamma^1\Gamma^2\Gamma^3\Gamma^4$ acts as the chirality matrix of $SO(4)$. Following [18], we make the truncation

$$\epsilon^- = \psi_\mu^- = \lambda_5^- = \lambda_\alpha^+ = 0. \quad (3.39)$$

ϵ^\pm satisfy $\Gamma^5\epsilon^\pm = \pm\epsilon^\pm$ with $\epsilon = \epsilon^+ + \epsilon^-$. We will now drop \pm superscript from ϵ , λ and ψ_μ .

In accordance with the bosonic truncation $T^{ij} = (T^{\alpha\beta}, T^{55}) = (X\tilde{T}^{\alpha\beta}, X^{-4})$, we truncate the $SL(5, \mathbb{R})/SO(5)$ coset representative as $\Pi_i^{\hat{i}} = (\Pi_\alpha^R, \Pi_5^{\hat{5}})$. With the identification $\Pi_\alpha^R = X^{-\frac{1}{2}}\mathcal{V}_\alpha^R$ and $\Pi_5^{\hat{5}} = X^2$, we can write $\tilde{T}^{\alpha\beta}$ in term of $SL(4, \mathbb{R})/SO(4)$ coset representative \mathcal{V}_α^R as

$$\tilde{T}^{\alpha\beta} = (\mathcal{V}^{-1})_R^\alpha (\mathcal{V}^{-1})_S^\beta \delta^{RS} \quad \text{and} \quad \tilde{T}_{RS} = (\mathcal{V}^{-1})_R^\alpha (\mathcal{V}^{-1})_S^\beta \delta_{\alpha\beta}. \quad (3.40)$$

We then find that equations (3.11) and (3.12) become

$$\begin{aligned}\delta\psi_\mu &= D_\mu\epsilon - \frac{1}{20}g(X\tilde{T}_{RR} + X^{-4})\gamma_\mu\epsilon - \frac{1}{40\sqrt{2}}X^{-1}(\gamma_\mu^{\nu\rho} - 8\delta_\mu^\nu\gamma^\rho)\Gamma_{RS}F_{\nu\rho}^{RS}\epsilon \\ &\quad - \frac{1}{60}X^{-2}\left(\gamma_\mu^{\nu\rho\sigma} - \frac{9}{2}\delta_\mu^\nu\gamma^{\rho\sigma}\right)S_{\nu\rho\sigma}^5\epsilon,\end{aligned}\tag{3.41}$$

$$\begin{aligned}\delta\lambda_R &= \frac{1}{4}\gamma^\mu\Gamma_RX^{-1}\partial_\mu X\epsilon + \frac{1}{2}\Gamma^S\gamma^\mu P_{RS}\epsilon + \frac{1}{16\sqrt{2}}X^{-1}\gamma^{\mu\nu}\left(\Gamma_{ST}\Gamma_R - \frac{1}{5}\Gamma_R\Gamma_{ST}\right)F_{\mu\nu}^{ST}\epsilon \\ &\quad - \frac{1}{10}gX^{-4}\Gamma_R\epsilon - \frac{1}{2}gX\left(\tilde{T}_{RS} - \frac{1}{5}\tilde{T}_{TT}\delta_{RS}\right)\Gamma^S\epsilon - \frac{1}{120}X^{-2}\gamma^{\mu\nu\rho}\Gamma_RS_{\mu\nu\rho}^5\epsilon.\end{aligned}\tag{3.42}$$

The constraint $\Gamma^i\lambda_i = 0$ imposes the condition $\lambda_5^+ = -\Gamma^R\lambda_R^-$. Therefore, the independent fields will be ψ_μ and λ_R . This is the reason for excluding $\delta\lambda_5$ in the above equations. We then identify $\Gamma^R\lambda_R$ with χ and $\hat{\lambda}_R = \lambda_R - \frac{1}{4}\Gamma_R\Gamma^S\lambda_S$ with λ^r in (2.17). Note that $\hat{\lambda}_R$ has only three independent components due to the condition $\Gamma^R\hat{\lambda}_R = 0$.

With these and the odd-dimensional self-duality, we end up with, after some gamma matrix algebra,

$$\begin{aligned}\delta\psi_\mu &= D_\mu\epsilon - \frac{1}{20}gX\tilde{T}\gamma_\mu\epsilon - \frac{1}{40\sqrt{2}}X^{-1}(\gamma_\mu^{\nu\rho} - 8\delta_\mu^\nu\gamma^\rho)\Gamma_{RS}F_{\nu\rho}^{RS}\epsilon \\ &\quad - \frac{1}{20}gX^{-4}\gamma_\mu\epsilon - \frac{1}{480}X^2(3\gamma_\mu^{\nu\rho\sigma\tau} - 8\delta_\mu^\nu\gamma^{\rho\sigma\tau})F_{\nu\rho\sigma\tau}\epsilon,\end{aligned}\tag{3.43}$$

$$\begin{aligned}\delta\chi &= X^{-1}\gamma^\mu\partial_\mu X\epsilon - \frac{2}{5}gX^{-4}\epsilon + \frac{1}{10}gX\tilde{T}_{RR}\epsilon \\ &\quad - \frac{1}{120}X^2\gamma^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}\epsilon - \frac{1}{20\sqrt{2}}X^{-1}\gamma^{\mu\nu}\Gamma_{RS}F_{\mu\nu}^{RS}\epsilon,\end{aligned}\tag{3.44}$$

$$\begin{aligned}\delta\hat{\lambda}_R &= -\frac{1}{2}\gamma^\mu\Gamma^SP_{\mu RS}\epsilon - \frac{1}{8}gX\tilde{T}_{SS}\Gamma_R\epsilon + \frac{1}{2}gX\tilde{T}_{RS}\Gamma^S\epsilon \\ &\quad - \frac{1}{8\sqrt{2}}X^{-1}\gamma^{\mu\nu}\Gamma_S\left(F_{\mu\nu}^{RS} + \frac{1}{2}\epsilon_{RSTU}F_{\mu\nu}^{TU}\right)\epsilon.\end{aligned}\tag{3.45}$$

In the above equations, we have used the following definitions

$$\begin{aligned}P_{RS} &= (\mathcal{V}^{-1})_{(R}^\alpha\left(\delta_\alpha^\beta d + gA_{(1)\alpha}^\beta\right)\mathcal{V}_\beta^T\delta_{S)T}, \\ Q_{RS} &= (\mathcal{V}^{-1})_{[R}^\alpha\left(\delta_\alpha^\beta d + gA_{(1)\alpha}^\beta\right)\mathcal{V}_\beta^T\delta_{S]T}, \\ D\epsilon &= d\epsilon + \frac{1}{4}\omega_{ab}\gamma^{ab} + \frac{1}{4}Q_{RS}\Gamma^{RS}.\end{aligned}\tag{3.46}$$

Notice that with our convention for $\Gamma^5\epsilon = \epsilon$, Γ_{RS} is anti-self dual. The field strength $F_{(2)}^{RS}$ appearing in (3.43) and (3.44) must be accordingly anti-self dual. This should be

identified with the $SU(2)$ field strength $F_{(2)}^i$ in (2.15) and (2.16). On the other hand, the self dual part of $F_{(2)}^{RS}$ appears in (3.45) and should be identified with $F_{(2)}^r$ in (2.17).

In more detail, after using gamma matrix identities such as $\gamma_\mu \gamma^{\nu\rho} = \gamma_\mu^{\nu\rho} + 2\delta_\mu^{[\nu} \gamma^{\rho]}$, we can rewrite equation (2.15) as

$$\begin{aligned} \delta\psi_\mu = & 2D_\mu\epsilon - \frac{\sqrt{2}}{30}e^{-\frac{\sigma}{2}}C\gamma_\mu\epsilon - \frac{1}{120\sqrt{2}}e^{-\sigma}H_{\rho\sigma\lambda\tau}(3\gamma_\mu^{\rho\sigma\lambda\tau} - 8\delta_\mu^\rho\gamma^{\sigma\lambda\tau})\epsilon \\ & - \frac{i}{10}e^{\frac{\sigma}{2}}F_{\rho\sigma}^i\sigma^i(\gamma_\mu^{\rho\sigma} - 8\delta_\mu^\rho\gamma^\sigma)\epsilon - \frac{4}{5}he^{2\sigma}\gamma_\mu\epsilon. \end{aligned} \quad (3.47)$$

Using the relation $C = -\frac{3}{2\sqrt{2}}g_1\tilde{T}$ given in [15] with the relation $g_2 = g_1 = -2g$ and identifying $F_{RS}\Gamma^{RS} = 2\sqrt{2}iF^i\sigma^i$, we find that equation (3.43) matches with (3.47). Similarly, equation (3.44) matches with (2.16). Note that in order to match the gravitino variation, we need to multiply (3.43) by two.

Comparing (2.17) and (3.45) is more complicated since various terms are not related to each other in a simple way. For example, we should write the anti-self dual part of Γ_{RS} in terms of the anti-self dual t' Hooft symbols $\bar{\eta}_{RS}^i$ and Pauli matrices σ^i

$$\Gamma_{RS}^{(-)} = i\sigma^i\bar{\eta}_{iRS} \quad (3.48)$$

and similarly for the self dual part

$$\Gamma_{RS}^{(+)} = i\sigma^r\eta_{rRS}. \quad (3.49)$$

Accordingly, we should identify

$$F^i = \frac{1}{2}\bar{\eta}_{RS}^i F^{RS} \quad \text{and} \quad F^r = \frac{1}{2}\eta_{RS}^r \left(F_{\mu\nu}^{RS} + \frac{1}{2}\epsilon_{RSTU}F_{\mu\nu}^{TU} \right). \quad (3.50)$$

Equation (3.45) should then match with (2.17), but we refrain from giving the full detail here due to the complicated algebra.

4. Embedding seven-dimensional RG flow to eleven dimensions

In this section, we will use the reduction ansatz obtained in the previous section to uplift some seven-dimensional solutions. The dimensional reduction gives rise to the condition $g_2 = g_1$. This makes the supersymmetric AdS_7 critical point with $SO(3)_{\text{diag}}$ symmetry found in [16] disappears. Accordingly, the flow solution given in [16] cannot be uplifted to eleven dimensions with the present reduction ansatz. However, to give examples of the uplifted solutions, we will study other solutions in the case of $g_2 = g_1$.

4.1 Uplifting AdS_7 solutions

We now further truncate the nine scalars given by $\tilde{T}_{\alpha\beta}$ to one scalar invariant under $SO(3)_{\text{diag}} \subset SO(3) \times SO(3) \sim SO(4)$. This scalar sector has already been studied in [16]. We will give more solutions in this section. Under $SO(3)_{\text{diag}}$, the nine scalars transform as $\mathbf{1} + \mathbf{3} + \mathbf{5}$. There is only one singlet. It can be checked that the $SO(3)_{\text{diag}}$ singlet correspond to

$$\mathcal{V}_\alpha^R = \begin{pmatrix} e^{\frac{\phi}{2}} \\ & e^{\frac{\phi}{2}} \\ & & e^{\frac{\phi}{2}} \\ & & & e^{-\frac{3\phi}{2}} \end{pmatrix} \quad \text{or} \quad \tilde{T}_{\alpha\beta} = \begin{pmatrix} e^\phi & & & \\ & e^\phi & & \\ & & e^\phi & \\ & & & e^{-3\phi} \end{pmatrix}. \quad (4.1)$$

$\tilde{T}_{\alpha\beta}$ can be written more compactly as $\tilde{T}_{\alpha\beta} = (\delta_{ab}e^\phi, e^{-3\phi})$ for $a, b = 1, 2, 3$. By using (3.34) and the explicit form of Γ^I and η^A given in [15], it is easy to verify that this \mathcal{V} precisely gives the $SO(3, 3)/SO(3) \times SO(3)$ coset representative L used in [16].

Using this and the relation $X = e^{-\frac{\sigma}{2}}$, we find the scalar potential

$$V = \frac{1}{2}g^2e^{-\sigma} \left[e^{5\sigma} + e^{-6\phi} - 6e^{-2\phi} - 3e^{2\phi} - 2e^{\frac{5}{2}\sigma-3\phi} (1 + 3e^{4\phi}) \right]. \quad (4.2)$$

This potential admits two AdS_7 critical points given by

$$\sigma = \phi = 0, \quad V_0 = -480h^2 \quad (4.3)$$

$$\sigma = -\frac{1}{10} \ln 2, \quad \phi = -\frac{1}{4} \ln 2, \quad V_0 = -160 \times 2^{\frac{3}{5}} h^2 \quad (4.4)$$

where we have used $g = 8h$ or equivalently $g_1 = -16h$ as given in [16]. By using the BPS equations given in [16], which are repeated below, we see that the second critical point is non-supersymmetric. Scalar masses at this critical point can be computed to be

$SO(3)_{\text{diag}}$	$m^2 L^2$
1	-12
1	12
3	0
5	-12

where the AdS_7 radius is given by $L = \sqrt{-\frac{15}{V_0}}$. The three massless scalars are the expected Goldstone bosons corresponding to the symmetry breaking of $SO(4)$ to $SO(3)$. One of the **1** and **5** scalars have masses below the BF bound $m^2 L^2 = -9$, so this critical point is unstable.

The first critical point is the trivial point preserving all supersymmetries and the full $SO(4)$ gauge symmetry. The scalar masses can be found in [16]. We will now uplift this AdS_7 vacuum to eleven dimensions. We begin with the coordinates $\mu^\alpha = (\cos \psi \hat{\mu}^a, \sin \psi)$ in which $\hat{\mu}^a \hat{\mu}^a = 1$. Since $\sigma = \phi = 0$, we then find $\Delta = 1$ and

$$ds_{11}^2 = e^{\frac{2r}{L_{UV}}} dx_{1,5}^2 + dr^2 + \frac{1}{32h^2} \left[d\xi^2 + \frac{1}{4} \cos^2 \xi (d\psi^2 + \cos^2 \psi d\Omega_2^2) \right] \quad (4.5)$$

$$\hat{F}_{(4)} = -\frac{3}{256h^3} \cos^3 \xi d\xi \wedge \epsilon_{(3)} \quad (4.6)$$

where $d\Omega_2^2$ is the metric on the two-sphere. The eleven dimensional geometry is given by $AdS_7 \times S^4$. Turning on the dilaton σ would deform the four-sphere but leave the S^3 inside invariant. If $\phi, \sigma \neq 0$, the metric would be further deformed in such a way that the S^2 part described by $d\Omega_2^2$ is invariant. The unbroken symmetry in this case is the $SO(3)$ isometry of this S^2 identified with the unbroken $SO(3)_{\text{diag}}$. The $SO(3)$ critical point is however unstable. Therefore, we will not consider AdS_7 solution with $SO(3)$ symmetry.

4.2 Uplifting RG flows to non-conformal $SO(3)$ Super Yang-Mills

To give more examples, we will study RG flow solutions to non-conformal Super Yang-Mills theories in the IR. We will work in the theory of section 2. With $g_2 = g_1$ and the standard domain wall metric ansatz $ds_7^2 = e^{A(r)} dx_{1,5}^2 + dr^2$, the BPS equations taken from [16] become

$$\phi' = -4e^{-\frac{\sigma}{2}-3\phi} (e^{4\phi} - 1) h, \quad (4.7)$$

$$\sigma' = \frac{8}{5} e^{-\frac{\sigma}{2}-3\phi} \left(1 + 3e^{4\phi} - 4e^{\frac{5}{2}\sigma+3\phi} \right) h, \quad (4.8)$$

$$A' = \frac{4}{5} e^{-\frac{\sigma}{2}-3\phi} \left(1 + 3e^{4\phi} + e^{\frac{5}{2}\sigma+3\phi} \right) \quad (4.9)$$

in which $\frac{d}{dr}$ is denoted by $'$. After changing to the new coordinate \tilde{r} given by $\frac{d\tilde{r}}{dr} = e^{-\frac{\sigma}{2}}$, we find the solution

$$16h\tilde{r} = \ln \left[\frac{1+e^\phi}{1-e^\phi} \right] - 2 \tan^{-1} \phi + C_1, \quad (4.10)$$

$$\sigma = \frac{2}{5} \left[\phi - \ln [1 + 12C_2 - 12C_2 e^{4\phi}] \right], \quad (4.11)$$

$$A = \frac{1}{4} \left[\phi - 2 \ln(1 - e^{4\phi}) \right] - \frac{1}{8} \sigma. \quad (4.12)$$

The solution interpolates between an AdS_7 in the UV, $\tilde{r} \sim r \rightarrow \infty$, and a domain wall in the IR, $4h\tilde{r} \rightarrow \tilde{C}$, for a constant \tilde{C} .

At the UV, the solution becomes

$$\sigma \sim \phi \sim e^{-16hr} \sim e^{-\frac{4r}{L_{UV}}}, \quad A \sim 4hr \sim \frac{r}{L_{UV}}. \quad (4.13)$$

The eleven-dimensional metric is given by (4.5).

In the IR, we find that ϕ blows up as

$$\phi \sim -\ln(4h\tilde{r} - \tilde{C}) \quad (4.14)$$

for a constant \tilde{C} . The behaviour of σ depends on the value of the integration constant C_2 .

For $C_2 = 0$, we find

$$\sigma \sim -\frac{2}{5} \ln(4h\tilde{r} - \tilde{C}) \sim -\frac{1}{2} \ln(4hr - C) \quad (4.15)$$

where we have used the relation between \tilde{r} and r in the IR limit with C being another integration constant. The seven-dimensional metric is given by

$$ds_7^2 = (4hr - C)^2 dx_{1,5}^2 + dr^2. \quad (4.16)$$

For $C_2 \neq 0$, the solution becomes

$$\begin{aligned} \sigma &\sim \frac{6}{5} \ln(4h\tilde{r} - \tilde{C}) \sim \frac{3}{4} \ln(4hr - C), \\ ds_7^2 &= (4hr - C)^{\frac{3}{4}} dx_{1,5}^2 + dr^2. \end{aligned} \quad (4.17)$$

Both cases give $V \rightarrow -\infty$, so the solution is physical by the criterion of [24].

We now look at the eleven-dimensional geometry. For $C_2 = 0$ and $C_2 \neq 0$, the eleven dimensional metric is given respectively by

$$\begin{aligned} ds_{11}^2 &= (1 - \sin^2 \xi \cos^2 \psi)^{-\frac{1}{3}} \left[\left(\frac{14}{3} h \rho \right)^2 dx_{1,5}^2 + d\rho^2 \right] + \frac{1}{32h^2} (1 - \sin^2 \xi \cos^2 \psi)^{-\frac{2}{3}} \times \\ &\quad \left[\left(\frac{14}{3} h \rho \right)^{-\frac{27}{7}} \sin^2 \xi \cos^2 \psi d\xi^2 + \frac{1}{4} \sin \xi \sin(2\psi) \left(\frac{14}{3} h \rho \right)^{-\frac{1}{2}} d\psi d\xi \right. \\ &\quad \left. + \frac{1}{4} \left(\frac{14}{3} h \rho \right)^{-\frac{20}{7}} d\psi^2 + \frac{1}{4} \cos^2 \psi \left(\frac{14}{3} h \rho \right)^{\frac{10}{7}} d\Omega_2^2 \right], \end{aligned} \quad (4.18)$$

$$\begin{aligned} ds_{11}^2 &= (\cos \xi \cos \psi)^{-\frac{2}{3}} \left[\left(\frac{14}{3} h \rho \right)^{\frac{13}{14}} dx_{1,5}^2 + d\rho^2 \right] + \frac{1}{32h^2} (\cos \xi \cos \psi)^{-\frac{4}{3}} \times \\ &\quad \left[\left(\frac{14}{3} h \rho \right)^{\frac{17}{14}} (1 - \sin^2 \xi \cos^2 \psi) d\xi^2 - \frac{1}{4} \sin \xi \sin(2\psi) \left(\frac{14}{3} h \rho \right)^{\frac{7}{4}} d\xi d\psi \right. \\ &\quad \left. + \frac{1}{4} \cos^2 \xi \left(\frac{14}{3} h \rho \right)^{\frac{10}{7}} (\sin^2 \psi d\psi^2 + \cos^2 \psi d\Omega_2^2) \right] \end{aligned} \quad (4.19)$$

where $(\frac{14}{3}h\rho)^{\frac{6}{7}} = 4hr - C$.

As expected, when turning on ϕ and σ , the warped factors involve coordinates (ξ, ψ) . The S^4 is then deformed leaving the S^2 intact. If only $\sigma \neq 0$, the S^3 part of the internal metric would be invariant as pointed in [17]. The deformation with only $\phi \neq 0$ is not possible since the BPS equation for σ would imply $\phi = 0$ as pointed out in [16].

5. Conclusions

In this paper, we have constructed $N = 2$ $SO(4)$ gauged supergravity in seven dimensions with topological mass term. The resulting theory admit AdS_7 vacua and could be useful in the context of the AdS/CFT correspondence. The resulting reduction ansatz has been found by truncating the S^4 reduction leading to $N = 4$ $SO(5)$ gauged supergravity and can be used to uplift seven-dimensional solutions to eleven dimensions. We have also constructed new seven-dimensional RG flow solutions and uplifted the resulting solutions to eleven dimensions. The flows can be interpreted as deformations of the UV $N = (1, 0)$ SCFT in six dimensions with $SO(4)$ symmetry to non-conformal SYM with $SO(3)_{\text{diag}}$ symmetry. These deformations are driven by vacuum expectation values of dimension 4 operators. Additionally, the result of this paper can be used to uplift flows to $SO(2)$ non-conformal gauge theories studied in [16] for $g_2 = g_1$.

However, the RG flow between two supersymmetric AdS_7 critical points recently found in [16] cannot be uplifted by using the reduction ansatz constructed here. It would be interesting to find an embedding of this solution in 10 or 11 dimensions. It is also interesting to extend the reduction ansatz given here to non-compact gauge groups $SO(3, 1)$ and $SO(2, 2)$. The internal manifold should involve hyperbolic spaces $H^{3,1}$ and $H^{2,2}$, respectively. Other possible non-compact gauge groups are $SL(3, \mathbb{R})$, $SO(2, 1)$ and $SO(2, 2) \times SO(2, 1)$. It would be very interesting to find higher dimensional origins for these gauge groups as well. Finally, more insight to six-dimensional gauge theories might be gained from studying these seven-dimensional gauged supergravities via AdS_7/CFT_6 correspondence. We hope to come back to these issues in future works.

Acknowledgments

The author gratefully thanks Eric Bergshoeff for useful correspondences. This work is supported by Chulalongkorn University through Ratchadapisek Sompoch Endowment Fund under grant Sci-Super 2014-001. The author is also supported by The Thailand Research Fund (TRF) under grant TRG5680010.

References

- [1] M. Cvetič, H. Lu and C. N. Pope, “Consistent Kaluza-Klein sphere reductions”, Phys. Rev. **D62** (2000) 064028, arXiv: hep-th/0003286.
- [2] B. de Wit and H. Nicolai, “The consistency of the S^7 truncation in $D = 11$ supergravity”, Nucl. Phys. **B281** (1987) 211.
- [3] H. Natase, D. Vaman and P. van Nieuwenhuizen, “Consistency of the $AdS_7 \times S_4$ reduction and the origin of self-duality in odd dimensions”, Nucl. Phys. **B581** (2000) 179-239, arXiv: hep-th/9911238.
- [4] M. Cvetič, H. Lu, C. N. Pope, A. Sadrzadeh and T. A. Tran, “Consistent $SO(6)$ reduction of type IIB supergravity on S^5 ”, Nucl. Phys. **B586** (2000) 275-286, arXiv: hep-th/0003103.
- [5] J. M. Maldacena, “The large N limit of superconformal field theories and supergravity”, Adv. Theor. Math. Phys. **2** (1998) 231-252, arXiv: hep-th/9711200.
- [6] R. G. Leigh and M. Rozali, “The large N limit of the $(2, 0)$ superconformal field theory”, Phys. Letts **B431** (1998) 311-316, arXiv: hep-th/9803068.
- [7] M. Berkooz, “A supergravity dual of a $(1, 0)$ field theory in six dimensions”, Phys. Lett. **B437** (1998) 315-317, arXiv: hep-th/9802195.
- [8] C. Ahn, K. Oh and R. Tatar, “Orbifolds $AdS_7 \times S^4$ and six-dimensional $(0, 1)$ SCFT”, Phys. Lett. **B442** (1998) 109-116, arXiv: hep-th/9804093.
- [9] E. G. Gimon and C. Popescu, “The operator spectrum of the six-dimensional $(1, 0)$ theory”, JHEP 04 (1999) **018**, arXiv: hep-th/9901048.
- [10] S. Ferrara, A. Kehagias, H. Partouche and A. Zaffaroni, “Membranes and fivebranes with lower supersymmetry and their AdS supergravity duals”, Phys. Lett. **B431** (1998) 42-48, arXiv: hep-th/9803109.
- [11] P. K. Townsend and P. van Nieuwenhuizen, “Gauged seven-dimensional supergravity”, Phys. Lett. **B125** (1983) 41-46.
- [12] E. Bergshoeff, I. G. Koh and E. Sezgin, “Yang-Mills-Einstein supergravity in seven dimensions”, Phys. Rev. **D32** (1985) 1353-1357.
- [13] Y. J. Park, “Gauged Yang-Mills-Einstein supergravity with three index field in seven dimensions”, Phys. Rev. **D38** (1988) 1087.
- [14] A. Salam and E. Sezgin, “ $SO(4)$ gauging of $N = 2$ supergravity in seven-dimensions”, Phys. Lett. **B126** (1983) 295.

- [15] E. Bergshoeff, D. C. Jong and E. Sezgin, “Noncompact gaugings, chiral reduction and dual sigma model in supergravity”, *Class. Quant. Grav.* **23** (2006) 2803-2832, arXiv: hep-th/0509203.
- [16] P. Karndumri, “RG flows in 6D $N = (1, 0)$ SCFT from $SO(4)$ half-maximal gauged supergravity” , *JHEP* 06 (2014) **101**, arXiv: 1404.0183.
- [17] H. Lu and C. N. Pope “Exact embedding of $N = 1$, $D = 7$ gauged supergravity in $D = 11$ ”, *Phys. Letts.* **B467** (1999) 67-72, arXiv: hep-th/9906168.
- [18] M. Cvetič, G. W. Gibbons and C. N. Pope, “A String and M-theory Origin for the Salam-Sezgin Model”, *Nucl. Phys.* **B667** (2004) 164-180, arXiv: hep-th/0308026.
- [19] M. Cvetič, H. Lu, C. N. Pope, A. Sadrzadeh and T. A. Tran, “ S^3 and S^4 reductions of type IIA supergravity”, *Nucl. Phys.* **B590** (2000) 233-251, arXiv: hep-th/0005137.
- [20] H. Nastase, D. Vaman and P. van Nieuwenhuizen, “Consistent nonlinear KK reduction of 11d supergravity on $AdS_7 \times S_4$ and self-duality in odd dimensions”, *Phys. Lett.* **B469** (1999) 96-102, arXiv: hep-th/9905075.
- [21] H. Nastase and D. Vaman, “On the nonlinear KK reductions on spheres of supergravity theories”, *Nucl. Phys.* **B583** (2000) 211-236, arXiv: hep-th/0002028.
- [22] M. Pernici, K. Pilch and P. van Nieuwenhuizen, “Gauged maximally extended supergravity in seven dimensions”, *Phys. Lett.* **B143** (1984) 103.
- [23] M. Cvetič, M. J. Duff, P. Hoxha, James T. Liu, H. Lu, J. X. Lu, R. Martinez-Acosta, C. N. Pope, H. Sati and T. A. Tran, “Embedding AdS Black Holes in Ten and Eleven Dimensions”, *Nucl. Phys.* **B558** (1999) 96-126, arXiv: hep-th/9903214.
- [24] S. S. Gubser, “Curvature singularities: the good, the bad and the naked”, *Adv. Theor. Math. Phys.* **4** (2000) 679-745.